INDIAN STATISTICAL INSTITUTE

Probability Theory II: B. Math (Hons.) I
Semester II, Academic Year 2018-19
Midsem Exam

Teacher: Parthanil Roy

Date: 26/02/2019 Maximum Marks: 30 Duration: 10:00 am - 1:00 pm

Note:

- **Please write your name on your answer booklet.**
- **There are three problems each carrying 10 marks with a total of 30 marks.**
- **Show all your calculations and write explanations when needed.**
- **You may use any fact proved in the class but do not forget to quote the appropriate result.**
- **You are NOT allowed to use class notes, books, homework solutions, list of theorems, formulas etc. If you are caught using any, you will get a zero grade in this exam.**
- **While specifying the distribution of a random variable, please write down the value(s) of the parameter(s) along with the name of the distribution.**

1. A random variable X is said to follow Laplace distribution with parameters $\mu \in \mathbb{R}$ and $\tau \in (0, \infty)$ (denoted by $X \sim \text{Laplace}(\mu, \tau)$) if X has a probability density function
$$f_X(x) = \frac{1}{2\tau} \exp\left(-\frac{|x-\mu|}{\tau}\right), \quad x \in \mathbb{R}.$$
   (a) (4 marks) Write down, with proper justification, an algorithm to simulate a random variable
$$Z \sim \text{Laplace}(0, 1).$$
   (b) (4 + 2 = 6 marks) If $Z \sim \text{Laplace}(0, 1)$, find a probability density function of $W := |Z|$. What distributions does W follow?

2. (10 marks) Suppose $U_1 \sim N(0, 1)$, $U_2 \sim N(0, 1)$ and U_1, U_2 are independent. Find a probability density function of $V_1 = 3U_1 + 4U_2$.
[**Hint:** You may use the auxiliary random variable $V_2 = 4U_1 - 3U_2$.]

3. Suppose X_1, X_2, X_3 is a random sample from exponential distribution with parameter $\lambda = 1$. Find the joint probability density functions of the following random vectors:
   (a) (6 marks) Find a joint probability density function of the random vector
$$\big(X_{(1)},\, X_{(2)} - X_{(1)},\, X_{(3)} - X_{(2)}\big).$$
   (b) (2 marks) Are $X_{(1)}$ and $X_{(3)} - X_{(1)}$ independent? Please justify your answer.
   (c) (2 marks) What distribution does $X_{(1)}$ follow?

Wish you all the best